Chemistry 271, Section 22xx
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Your Name: _____

Your SID #: _____

General Chemistry and Energetics
Exam II (100 points total)

Your Section # or time: _____

November 1, 2010

You have 52 minutes for this exam.

Exams written in pencil or erasable ink will not be re-graded under any circumstances.

Explanations should be <u>concise</u> and <u>clear</u>. I have given you more space than you should need. There is extra space on the last page if you need it.

You will need a calculator for this exam. No other study aids or materials are permitted.

Generous partial credit will be given, *i.e.*, if you don't know, guess.

Useful Equations:

$$K_a = [H^+][A^-]/[HA]$$

$$pH = -\log([H^+])$$

$$K_b = [HA][HO^-]/[A^-]$$

$$K_w = [H^+][HO^-]$$

$$pH = pK_a + \log [A^-]/[HA]$$

$$\Delta G° = -RT\ln K_{eq}$$

$$R = 0.08206 \text{ L·atm/mole K}$$

$$0 °C = 273.15 \text{ K}$$

$$\ln K_{eq} = -\Delta H°/(RT) + \Delta S°/R$$

$$\Delta S - q/T \geq 0$$

$$R = 8.314 \text{ J/mole K} = 1.987 \text{ cal/mole K}$$

$$S = k\ln W$$

$$\Delta G = \Delta H - T\Delta S$$

$$E = \sum n_i \varepsilon_i$$

$$W = N!/(\prod n_i!)$$

$$n_i/n_0 = \exp[-(\varepsilon_i - \varepsilon_0)/kT]$$

$$N = \sum n_i$$

Chemical standard state: 1 M solutes, pure liquids, 1 atm gases

Biochemical standard state: pH 7, all species in the ionic form found at pH 7

<u>**Honor Pledge: At the end of the examination time, please write out the following sentence and sign it, or talk to me about it:**</u>

"I pledge on my honor that I have not given or received any unauthorized assistance on this examination."

1. (24 pts) Multiple choice: Circle the single best answer for each question

(a; 4 pts) A configuration in statistical thermodynamics is:

(1) A description of the position and velocity of each particle in a system.

(2) A set of microstates that all share the same set of observables P, T, n, etc.

(3) All of the possible microstates that have a given value of total E.

(4) All of the possible microstates that could describe a system at any value of E.

(b; 4 pts) The Boltzmann distribution describes:

(1) The distribution of energy among particles in the microstates of the predominant configuration.

(2) The distribution of microstates among different ensembles.

(3) The distribution of coin flips for small numbers of coins.

(4) The distribution of entropy among different microstates.

(c; 4 pts) Boltzmann defined entropy as $S = k_B \ln W$ because

(1) It's a postulate – it doesn't need to be justified.

(2) Using a logarithm is the only way to make entropy an extensive state function that increases as W increases.

(3) The value of "k_B" could be chosen to make the numerical values of S fall into an arbitrary and convenient range.

(4) All of the above.

(d; 4 pts) Exothermicity *per se* makes reactions tend to be favorable because

(1) It accompanies bond breaking

(2) It decreases the total energy of the universe

(3) It increases the entropy of the surroundings

(4) None of the above

(e; 4 pts) Protein folding from a random coil to a native tertiary structure occurs with

(1) Positive $\Delta H°$ and negative $\Delta S°$

(2) Positive $\Delta H°$ and positive $\Delta S°$

(3) Negative $\Delta H°$ and positive $\Delta S°$

(4) Negative $\Delta H°$ and negative $\Delta S°$

(f; 4 pts) The temperature at which $W = 1$ for a system and therefore $S = 0$ is

(1) Unknowable

(2) Absolute zero

(3) 100 °C

(4) Infinite

Score for the page_____

2.   (20 pts) Short-answer questions

(a; 12 pts) Briefly describe the ergodic hypothesis and why it is hard to test in practice.

(+4)   - The ergodic hypothesis hold that any microstate that has the same
total energy is as likely to occur as any other, so that if we wait
long enough all possible microstates will occur. Or - we observe   (+4)
a given configuration at random according to the properties of
the total microstates in the configuration.

(+4)   - It's hard to test because we can't actually observe microstates and
for either because we may not waht to wait the lifetime of the universe for
rare ones to show up.

(b; 8 pts) List one biotechnology technique that requires nucleic acid hybridization, and briefly and
qualitatively describe why/how nucleic acid thermodynamics can be applied to the design of the
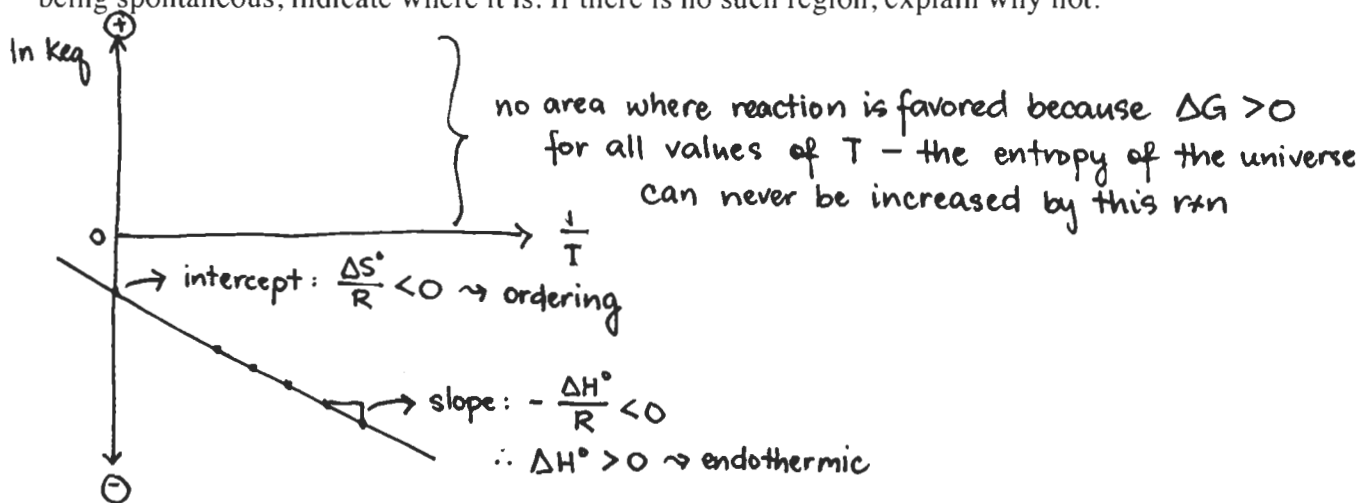oligonucleotides used.

   - DNA sequencing .

   - PCR   (+4) for any one, and there are
                                                  other correct answers
   - microarrays

   - prediction of RNA structure

   For example -

(+2)   We chose PCR/ sequencing primers according to the T_m desired, or we
Choose experimental conditions based on the predicted T_m. The sequences   (+2)
can be made longer or shorter or designed to complement a different area for any
according to the thermodynamics of perfect matches vs. mismatches   change

                                                                  Score for the page_____

3. (31 pts) Thermodynamics

(a; 12 pts) Sketch a van't Hoff plot for the temperature dependence of the equilibrium constant for an endothermic ordering reaction A+B -> C. If there is a region of the graph corresponding to the reaction being spontaneous, indicate where it is. If there is no such region, explain why not.

In k_{eq} (y-axis, $\oplus$ / $\ominus$)

$\frac{1}{T}$ (x-axis)

no area where reaction is favored because $\Delta G > 0$ for all values of T — the entropy of the universe can never be increased by this r×n

intercept: $\frac{\Delta S°}{R} < 0 \rightsquigarrow$ ordering

slope: $-\frac{\Delta H°}{R} < 0$

$\therefore \Delta H° > 0 \rightsquigarrow$ endothermic

(+1) (×) for idea of a straight line plot

(+1) for each axis label

(+2) for $\ominus$ intercept on y-axis

(+2) for $\ominus$ slope

(+3) for stating there is no area where reaction is favorable

(+3) for explanation in terms of $\oplus$ free energy change or failure to increase $S_{universe}$

Score for the page_____

The chart on the left below shows one possible distribution of 48 quanta of energy among 24 molecules, similar to our discussion of microstates in class.
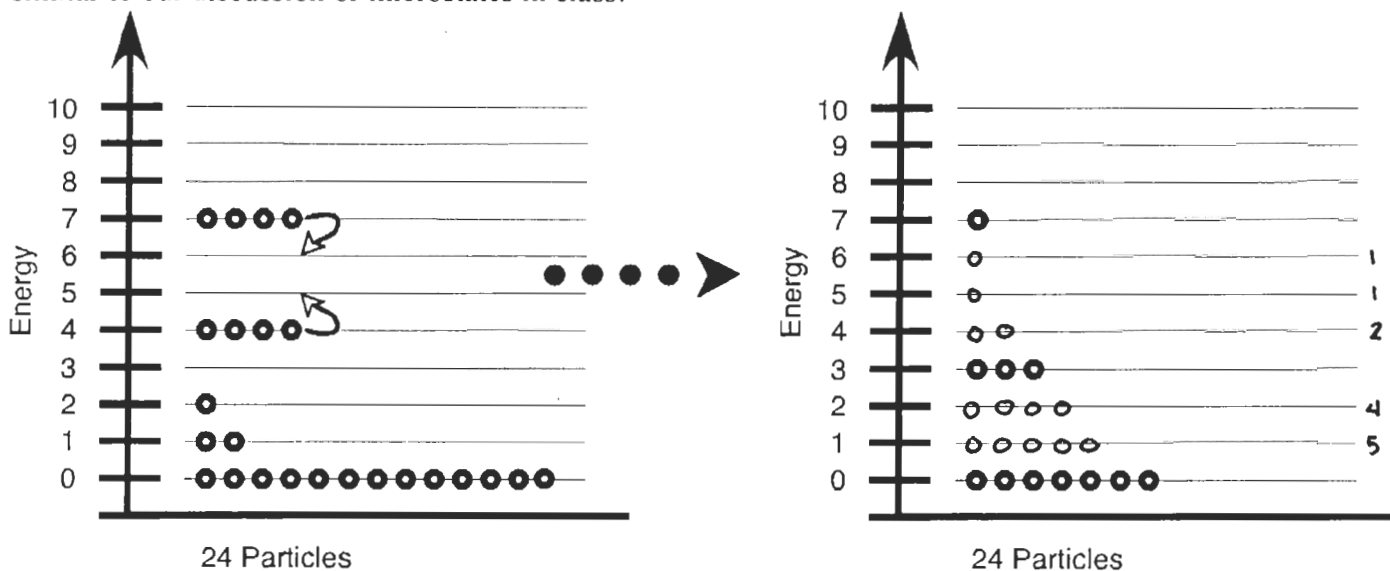
(b; 3 pts) Write down but do not evaluate a numerical expression for the number of microstates W for the configuration shown on the left above.

$$W = \frac{N!}{(n_0!)(n_1!)(n_2!)\cdots} = \frac{24!}{13!\,2!\,1!\,4!\,4!} = \frac{24 \cdot 23 \cdot 22 \cdot \ldots \cdot 14}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$$

(c; 6 pts) Now, consider transferring one quantum of energy from one of the molecules with 7 quanta to one of the ones with 4, as indicated by the arrows in the chart on the left above. This will yield molecules with 5 and 6 quanta. Write down the expression for W for this new configuration. By what factor did W increase upon this small redistribution of energy? You do not need to evaluate W to figure this out.

$$W_2 = \frac{24!}{13!\,2!\,1!\,3!\,1!\,1!\,3!} \qquad \text{(+3)}$$

$$\frac{W_2}{W_1} = \frac{13!\,2!\,1!\,4!\,4!}{13!\,2!\,1!\,3!\,1!\,1!\,3!} = 4 \times 4 = 16 \qquad \text{(+3)}$$

Score for the page_____

(d; 5 pts) If we continue to redistribute energy among the particles, we will reach the equilibrium state described by the Boltzmann distribution. By drawing circles in the chart on the right above, add in a reasonable guess for the energy levels of the 13 remaining molecules according to Boltzmann,.

$$13 = 5 + 4 + 2 + 1 + 1 \quad (\text{best answer})$$
$$= 6 + 3 + 2 + 1 + 1$$
$$= 4 + 4 + 3 + 1 + 1$$
$$= 5 + 3 + 2 + 2 + 1$$

all are acceptable — any decreasing set of 5 numbers that add up to 13

(e; 5 pts) Clearly W would be maximized if every n_i were equal to 1 or 0. Why doesn't every isolated system just evolve to maximum entropy by placing every molecule on its own unique level? Write down the equation from the front page that expresses this constraint.

(+3) — The energy has to come from somewhere — at low T / low E, there just isn't enough energy and most of the particles will be in the ground state.
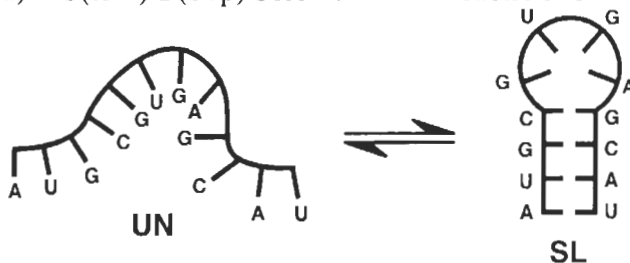
(+2) $\sum \varepsilon_i n_i = E$

4.   (25 pts) Applied thermodynamics: RNA folding

The folding of a single-stranded RNA into a stem-loop is simpler than hybridization of two separate strands to make a duplex. Consider the equilibrium UN(folded) ⇌ S(tem) L(oop) below. For the reaction as written,

$\Delta H° = -52000$ J/mol and
$\Delta S° = -150$ J/mol K
(made up but reasonable numbers).

UN   SL

(a; 3 pts) Calculate the free energy change $\Delta G°$ at 37 °C.

$$\Delta G° = \underbrace{\Delta H° - T\Delta S°}_{(+1)} = \underbrace{-52000 + (273.15 + 37)(150)}_{(+1)} = -5478 \text{ J/mol}$$

$$\text{or} = -5500 \text{ J/mol}$$

(+1) for either

(b; 6 pts) Write down the equilibrium constant expression and calculate its value at 37 °C.

$$\underbrace{K_{eq} = \frac{[SL]}{[UN]}}_{(+2)} = \underbrace{e^{-\frac{\Delta G°}{RT}}}_{(+2)} = e^{\frac{5478}{8.314 \times 310.15}} = \underbrace{8.37}_{(+2)}$$

Score for the page_____

(c; 6 pts) Calculate the percentage of the RNA in the SL form at 37 °C.

$$\frac{[SL]}{[UN]} = 8.37 \quad \text{so} \quad \frac{[SL]}{[UN] + [SL]} = \frac{8.37 \, [UN]}{9.37 \, [UN]} \times 100\% = 89\%$$

$$(+3) \qquad\qquad\qquad\qquad\qquad (+3)$$

(d; 2 pts) The melting temperature T_m is the temperature at which 50 % of the RNA is in the SL form and 50 % is in the UN form. What is the equilibrium constant for UN ⇌ SL at the T_m? (Note that this is not the same K_{eq} as for formation of duplex from separate strands, which we treated in lecture.)

$$K_{eq} = \frac{[SL]}{[UN]} = \frac{50}{50} = 1$$

(e; 2 pts) What is $\Delta G°$ at the T_m? (Again, differs from the form we saw for duplex formation)

$$\Delta G° = -RT_m \ln K = 0$$

(f; 6 pts) Calculate the T_m. (Again, differs from the form we saw for duplex formation)

$$\Delta G° = 0 = \Delta H° - T_m \Delta S° \quad (+2)$$

$$T_m = \frac{\Delta H°}{\Delta S°} = \frac{-52000 \, J/mol}{-150 \, J/mol\cdot K} = 347 \, K = 73.5 \, °C$$

$$(+2) \qquad\qquad\qquad\qquad (+2)$$

Page	Score
2	/24
3	/20
4	/12
5	/9
6	/19
7	/16
Total	

Score for the page_____